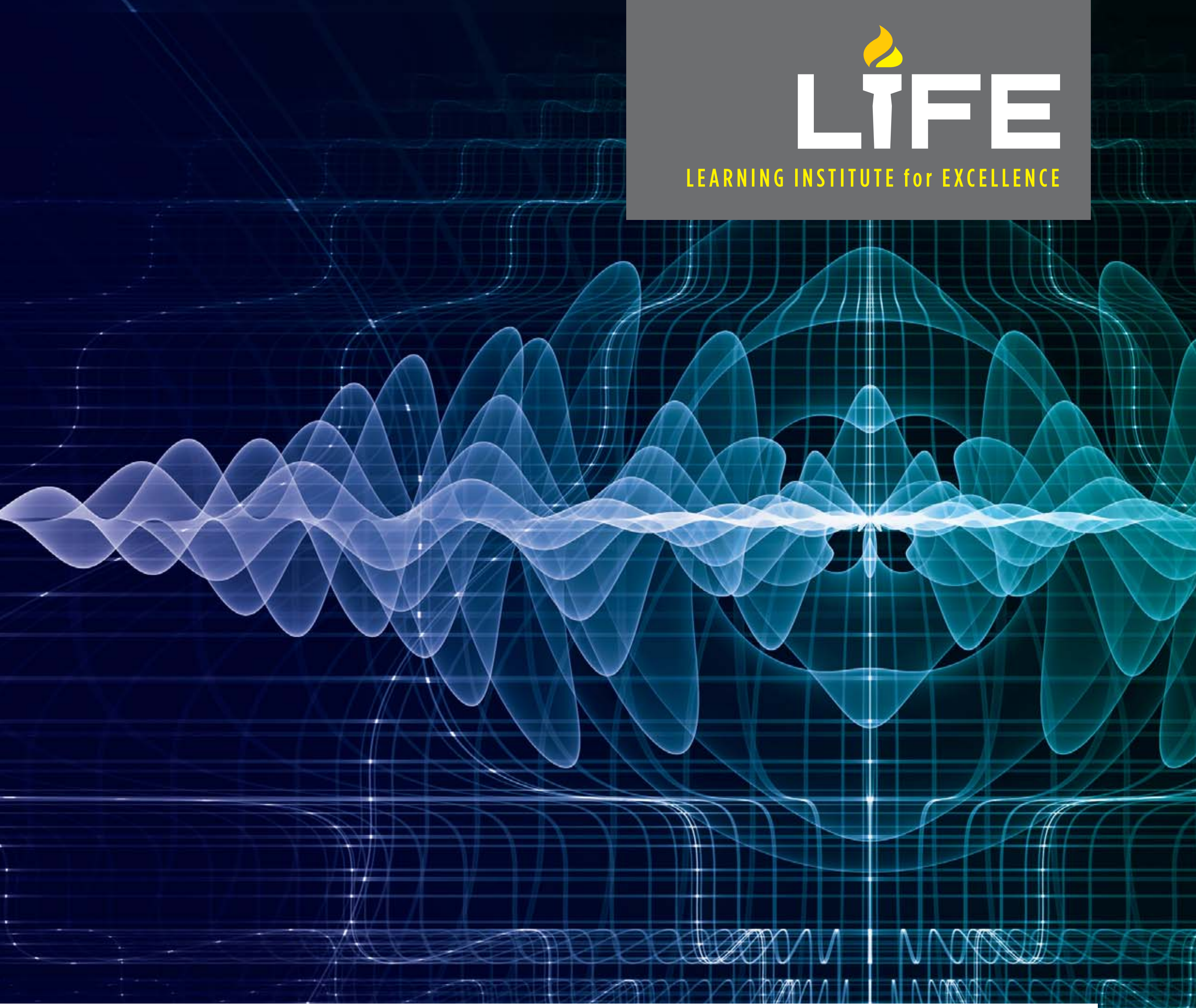

LIFE

LEARNING INSTITUTE for EXCELLENCE

Tools for Mastering **Mathematics**

For Teachers, Students, Curriculum Writers, Administrators and Those Helping Students Excel in Mathematics and Beyond

Amos Tarfa & Nathan Jersett

IGNITE LIFELONG LEARNING

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The Learning Institute For Excellence Presents:

THE NORTHLAND STEM ACADEMY ONLINE PROGRAM

Math Courses Available Starting January 2020

The Inspirational Mathematics Program is a subscription based program from the Learning Institute For Excellence (LIFE). It comes with certain tools which can be used together to help students be the best they can be in Mathematics. We believe that people can do Mathematics with excellence and that they can enjoy it in the process.

The Program is broken into 3 Courses:

Tools for Mastering Mathematics Course 1: Pre-Algebra & Fundamentals of Algebra
Tools for Mastering Mathematics Course 2: Algebra 2 & PreCalculus
Tools for Mastering Mathematics Course 3: Trigonometry & Geometry

Course	Details
1. Tools for Mastering Mathematics Course 1 (will complete Pre-Algebra & most of Algebra 1 Coursework)	This covers concepts in Pre-Algebra and introduces students to the fundamentals of Algebra and Geometry formulas
2. Tools for Mastering Mathematics Course 2 (will complete Algebra 2/ Pre-Calculus Coursework)	This covers concepts in Algebra 2 and prepares students for more in-depth Geometry, Pre-Calculus and Standardized Exams such as the ACT
3. Tools for Mastering Mathematics Course 3(will complete Trigonometry & Geometry Coursework)	This covers concepts in Geometry Proofs, Trigonometry and more in-depth concepts in Math and Math History. Students will be able to take the CLEP Pre-Calculus Course after this course

The Courses can be found on our Teachable Page. They take students through 5 Credits in Mathematics: Pre-Algebra, Algebra 1, Algebra 2, Pre-Calculus and Geometry.
Students can start taking these courses once they are ready for Pre-Algebra. College students who need to review concepts can also use these programs.

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"What students need fundamentally in academics today is not information, they need inspiration. They can get information from the library as well as google. Once they are inspired, the technology and coaching/teaching available will help them excel in whatever they choose to study. With the technology and tools available, we can help students master concepts in Mathematics like never before. We need to constantly seek ways to get them to care about excelling in learning and all areas of LIFE."

-Amos Tarfa [Founder, Learning Institute For Excellence (LIFE)] Math & Education Consultant

This program provides an opportunity for students to keep the passion for Mathematics through studying the masters and the beauty of mathematics. At the end of this series, students will have refined study skills and tools to excel in Calculus and beyond.

By getting this subscription students have access to Math tutors and coaches through the Learning Institute For Excellence through e-mail, and video conference calls using Zoom.

BOOKS AND SUPPLIES:
Students will be given books and resources which they can use for their studies. s
Students will be given the books and resources for the course at the appropriate time:

1. Tools For Mastering Mathematics by Amos Tarfa and Nathan Jersett
2. Redwoods PreAlgebra Textbook
3. Tyler Wallace Beginning and Intermediate Algebra Text
4. Larson PreCalculus Textbook
5. Geometry by Alexander Koeberlein.
6. Optional: Journey Through Genius
7. Optional: A Copy of Leonhard Euler's Elements of Algebra
8. Optional: World of Mathematics Math History Book
9. Optional: ACT 36 Guidebook
10. Optional: Excellence in Academics by Amos Tarfa and WPI Study Skills Handbook.

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TOOLS FOR MASTERING MATHEMATICS REFERENCE BOOKLET FOR STUDENTS

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Part 2 Table of Contents

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This is a tool students can always refer back to. It is important that the students recognize the interconnectivity of math topics.

They can use this guide as they study for the ACT and SAT as well.

The concepts are from the following courses:

Pre-Algebra
Algebra 1
Algebra 2
Pre-Calculus
Basic Probability and Statistics

Place Value

What is the difference between 1,000,000 and 1? It is obvious that they look different, but those 0's mean a lot. In fact, the difference is understood by using place value. Each 0 and each 1 means something in the place that you find it. That is because each place the number is put is understood to have a value.

Since our number system is base 10, each place that is different from the exact center, the decimal place or the ones place, will be some power of 10. Furthermore, the factors of 10 that are in a number may be multiplied by one of the 10 digits our number system uses: 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9.

Every number we write is actually a combination of digits multiplied by factors of 10. We can, therefore, break up the number back into its parts. To do this, you need to understand the different powers of ten that are within the place value. For now, lets look at a few powers of ten within a number.

What are the place values in the number 1,526,049?

Number	Place Value	Power of Ten
1	Million	1,000,000
5	Hundred Thousand	100,000
2	Ten Thousand	10,000
6	Thousand	1,000
0	Hundred	100
4	Ten	10
9	One	1

So, the number 1,526,049 is the same as $1 \cdot 1,000,000 + 5 \cdot 100,000 + 2 \cdot 10,000 + 6 \cdot 1,000 + 0 \cdot 100 + 4 \cdot 10 + 9 \cdot 1$.

The simple operations you do on numbers–addition, subtraction, multiplication, and division–all deal with changing the place values. When I say 15 + 26, I actually mean two 10s and six 1s in addition to one 10 and five 1s. How do we add it? Well, count how many ones you have now, but don't go above nine 1s (since this is the largest digit we have). When we have nine 1s and have an additional 1, then we move this whole group into the 10s place value, as $9 + 1 = 10$. So, in our example, we have nine 1s and two 1s, so we take the nine and one of the two and make a 10 out of it, leaving us with just one 1 left. Then, we count how many 10s there are. We have two 10s and added one 10, as well as getting an additional 10 from the 1s. This gives us four 10s, and a final count of four 10s with one 1. This is 41.

What does a multiplication problem like $3 \cdot 25$ mean? Well, it is saying, we have 3 different quantities of 25. Each quantity of 25 is two 10s and five 1s, so adding the 1s together gives a set of 10 and five 1s. Then adding the 10s together gives seven 10s. The grand total is 75.

Decimal Place Value

What are the place values of decimals? Let's look at 12,345.56789. We have already talked about the place values of 12,345, these follow the normal rules, but what about the .56789 portion?

Number	Place Value	Power of Ten
5	Tenths	0.1
6	Hundredths	0.01
7	Thousandths	0.001
8	Ten Thousandths	0.0001
9	Hundred Thousandths	0.00001

You can break up the decimal portion just as easily as the rest of the numbers, in the same way. More place value names and powers of ten are shown in the next section.

Powers of Ten

Powers of Ten are not only important in understanding the place value, but also for writing in scientific notation, using the Metric System, quickly reducing fractions, and more.

...	...	...
Billionth	10^{-9}	0.000000001
Hundred Millionth	10^{-8}	0.00000001
Ten Millionth	10^{-7}	0.0000001
Millionth	10^{-6}	0.000001
Hundred Thousandth	10^{-5}	0.00001
Ten Thousandth	10^{-4}	0.0001
Thousandth	10^{-3}	0.001
Hundredth	10^{-2}	0.01
Tenth	10^{-1}	0.1
One	10^{0}	1
Ten	10^{1}	10
Hundred	10^{2}	100
Thousand	10^{3}	1,000
Ten Thousand	10^{4}	10,000
Hundred Thousand	10^{5}	100,000
Million	10^{6}	1,000,000
Ten Million	10^{7}	10,000,000
Hundred Million	10^{8}	100,000,000
Billion	10^{9}	1,000,000,000
Trillion	10^{12}	1,000,000,000,000
Quadrillion	10^{15}	1,000,000,000,000,000
Quintillion	10^{18}	1,000,000,000,000,000,000
...	...	...
Googol	10^{100}	10,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000
...	...	...
Googolplex	$10^{10^{100}}$	...
...	...	...

Rounding and Estimation

Rounding

We round to make numbers easier to use. The way to round is to find the place value you want to round, then look at the number to the right. If that number is 5 or higher add one to the number in the place value you want then change all of the numbers to the right of this place to 0. If the number is 4 or less, then just change all of the numbers to the right of the place to 0.

Example 1:
What do we get if we round 363,755 to the hundreds place? The hundreds place is the 7 in this number. Then, to round this number, we need to look at the number to the right of the 7: the number 5. Is this number 5 or higher? Yes, so we need to add one to the number 7 (giving 8) and change all of the numbers to the right to 0. Doing this gives 363,800.

Example 2:
What do we get if we round 4,567.823 to the tenths place? The tenths place is the 8 in this number. So, the number to the right of this place is a 2, which is 4 or less. This means that we do nothing but change all numbers to the right of this place to 0. This gives the number 4,567.800 = 4,567.8. If there are zeros after the last nonzero digit, then it is acceptable to drop them, but only if they are to the right of the decimal. This is due to the fact that they will not change the place values of other digits.

Estimation

Sometimes we need to make numbers more manageable by estimating their value. This can be done by either knowing their exact value and rounding to a certain spot or picking what a number is by an educated guess. Estimation is used to quickly error check certain calculations, quickly do calculations by making the numbers easier to handle, and even just make numbers easier to remember. Think about it, which is easier to remember: 3 million or 3,124,645.897?

Estimation problems can be really easy to find, and as you handle more and more problems using estimation, you will be a better judge of what estimations are good guesses or bad guesses. Let's do a simple estimation problem, how many ice cream cones will you have eaten in your lifetime? To make it easier, let's assume you live until 100 years old. Even though you may have had more ice cream cones when you are younger, you probably eat less when you are older, and even though you probably eat a lot of cones during the summer, you will eat less when you are in winter. So, balancing it all to one amount per year, let's say you have 20 ice cream cones per year, every year of your life. Then, all you need to do is $100 \cdot 20$ which is 2000 ice cream cones. Of course, depending on the person and lifestyle, this can change, but it does a decent job giving a general number.

Addition Table

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27
8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28
9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29
10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32
13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33
14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34
15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35
16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36
17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37
18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38
19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39
20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40

Table 1: Addition Table from 1 - 20

Multiplication Table

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
2	2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34	36	38	40
3	3	6	9	12	15	18	21	24	27	30	33	36	39	42	45	48	51	54	57	60
4	4	8	12	16	20	24	28	32	36	40	44	48	52	56	60	64	68	72	76	80
5	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95	100
6	6	12	18	24	30	36	42	48	54	60	66	72	78	84	90	96	102	108	114	120
7	7	14	21	28	35	42	49	56	63	70	77	84	91	98	105	112	119	126	133	140
8	8	16	24	32	40	48	56	64	72	80	88	96	104	112	120	128	136	144	152	160
9	9	18	27	36	45	54	63	72	81	90	99	108	117	126	135	144	153	162	171	180
10	10	20	30	40	50	60	70	80	90	100	110	120	130	140	150	160	170	180	190	200
11	11	22	33	44	55	66	77	88	99	110	121	132	143	154	165	176	187	198	209	220
12	12	24	36	48	60	72	84	96	108	120	132	144	156	168	180	192	204	216	228	240
13	13	26	39	52	65	78	91	104	117	130	143	156	169	182	195	208	221	234	247	260
14	14	28	42	56	70	84	98	112	126	140	154	168	182	196	210	224	238	252	266	280
15	15	30	45	60	75	90	105	120	135	150	165	180	195	210	225	240	255	270	285	300
16	16	32	48	64	80	96	112	128	144	160	176	192	208	224	240	256	272	288	304	320
17	17	34	51	68	85	102	119	136	153	170	187	204	221	238	255	272	289	306	323	340
18	18	36	54	72	90	108	126	144	162	180	198	216	234	252	270	288	306	324	342	360
19	19	38	57	76	95	114	133	152	171	190	209	228	247	266	285	304	323	342	361	380
20	20	40	60	80	100	120	140	160	180	200	220	240	260	280	300	320	340	360	380	400

Table 2: Multiplication Table from 1 - 20

Order of Operations and Number Chart

Order of Operations: PEMDAS

1st: (P) Parentheses
2nd: (E) Exponents
3rd: (MD) Multiplication and Division (left to right)
4th: (AS) Addition and Subtraction (left to right)

Whenever you need to solve an expression that has only numbers, you always follow this order.

Number Chart:

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Properties of Real Numbers

Additive Identity:

$$5 + 0 = 5$$
$$a + 0 = a$$

Inverse Property of Addition:

$$2 + (-2) = 0$$
$$c + (-c) = 0$$

Multiplicative Identity:

$$9 \cdot 1 = 9$$
$$h \cdot 1 = h$$

Inverse Property of Multiplication:

$$4 \cdot \frac{1}{4} = 1$$
$$m \cdot \frac{1}{m} = 1$$

Zero Property of Multiplication:

$$38 \cdot 0 = 0$$
$$n \cdot 0 = 0$$

Commutative Property of Addition:

$$5 + 3 = 3 + 5 \quad \text{and both sides equal 8}$$
$$x + y = y + x$$

Commutative Property of Multiplication:

$$3 \cdot 4 = 4 \cdot 3 \quad \text{and both sides equal 12}$$
$$p \cdot r = r \cdot p$$

Associative Property of Addition:

$$(2 + 3) + 4 = 2 + (3 + 4) \quad \text{becomes}$$
$$5 + 4 = 2 + 7 \quad \text{and both sides equal 9}$$
$$(a + b) + c = a + (b + c)$$

Associative Property of Multiplication:

$$(2 \cdot 3) \cdot 4 = 2 \cdot (3 \cdot 4) \quad \text{becomes}$$
$$6 \cdot 4 = 2 \cdot 12 \quad \text{and both sides equal 24}$$
$$(a \cdot b) \cdot c = a \cdot (b \cdot c)$$

Distributive Property:

$$2 \cdot (5 + 6) = 2 \cdot 5 + 2 \cdot 6 \quad \text{becomes}$$
$$2 \cdot 11 = 10 + 12 \quad \text{and both sides equal 22}$$
$$a \cdot (b + c) = a \cdot b + a \cdot c$$

Integer Arithmetic

Integer Multiplication (remember, two numbers next to each other, in parentheses, are multiplying):

$$+a \cdot +b = +a \cdot b$$
$$+a \cdot -b = -a \cdot b$$
$$-a \cdot +b = -a \cdot b$$
$$-a \cdot -b = +a \cdot b$$

Examples:

$$2 \cdot 4 = 8$$
$$(-3)(5) = -15$$
$$-8 \cdot -2 = 16$$
$$7(-3) = -21$$

Integer Addition:

If both numbers are +, then just add them: 7 + 3 = 10

If the bigger number is + and the smaller number is -, then subtract like usual: 10 - 4 = 6

If the bigger number is - and the smaller number is +, then subtract the smaller from the bigger and give the answer a negative sign: -8 + 3 = ? Well, 8 - 3 = 5, then the answer is -5.

If both numbers are -, then add the numbers and give the answer a negative number: -4 - 6 = ? Well, 4 + 6 = 10, then the answer is -10.

Whenever two signs are right next to each other, follow the integer multiplication rules to see how the signs combine:

$$+(+ = ++ = +,$$
$$-(+ = -+ = -,$$
$$-(- = -- = +,$$
$$+(- = +- = -.$$

Examples:

$$-5 + 4 = 4 - 5 = -1$$
$$-6 + 8 = 8 - 6 = 2$$
$$12 - (-10) = 12 + 10 = 22$$
$$-5 - 1 = -6$$
$$2 + 8 = 10$$
$$-1 - (-6) = -1 + 6 = 5$$

Divisibility Rules

For number x to be divisible by number y, when you take $\frac{x}{y}$, you must get a whole number as a result, not a fraction or decimal.

2	Even numbers are divisible by 2. Even numbers are those that end in 0, 2, 4, 6, or 8.
3	Add the digits, if the addition is divisible by 3, then the original number is divisible by 3. 363? $3 + 6 + 3 = 12$, $12 \div 3 = 4$, Yes. 373? $3 + 7 + 3 = 13$, $13 \div 3 = 4.333\ldots$, No.
4	Look at the number in the last two digits spot, if this number is divisible by 4, then the original number is divisible by 4. 1,524? $24 \div 4 = 6$, Yes. 863? $63 \div 4 = 15.75$, No.
5	If the number ends with 0 or 5, then it is divisible by 5.
6	If the number is divisible by 2 and 3, then it is divisible by 6.
7	Double the last digit and subtract it from the number that remains, if the subtraction is 0 or divisible by 7 then the original number is divisible by 7. 98? $8 \cdot 2 = 16$, $9 - 16 = -7$, $-7 \div 7 = -1$, Yes. 164? $4 \cdot 2 = 8$, $16 - 8 = 8$, $8 \div 7 = 1.142\ldots$, No.
8	If the number in the last three digits spot is divisible by 8, then the original number is divisible by 8. 51,200? $200 \div 8 = 25$, Yes. 81,106? $106 \div 8 = 13.25$, No.
9	Add the digits, if the addition is divisible by 9, then the original number is divisible by 9. 864? $8 + 6 + 4 = 18$, $18 \div 9 = 2$, Yes. 1,502? $1 + 5 + 0 + 2 = 8$, $8 \div 9 = 0.888\ldots$, No.
10	If the number ends with 0, then it is divisible by 10.
11	If you sum up every second digit then subtract the rest of the digits added up, and if the answer is 0 or divisible by 11, then the original number is divisible by 11. 1,232? $2 + 2 - (1 + 3) = 0$, Yes. 653? $5 - (6 + 3) = -4$, $-4 \div 11 = 0.808\ldots$, No.
12	If the number is divisible by 3 and 4, then it is divisible by 12.

Prime Number Properties

Prime numbers are numbers that are only divisible by themselves and 1. The first prime number is 2, and we only consider numbers greater than 2 to be candidates to be prime numbers. To see if a number is prime, you need to see if this number is divisible by all of the previous prime numbers. This is an easy process for small, one to two digit numbers, but can be a hassle for larger numbers.

All whole numbers (excluding 0 and 1) can be factorized into prime numbers. In fact, every number has a unique factorization, meaning when you multiply different combinations of prime numbers together, you will get different numbers every single time.

Prime numbers example, between 2 and 100:
2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97

Since all even numbers are divisible by 2, then the only even number that can be prime is 2. The number 9 is not prime because $9 \div 3 = 3$. The number 15 is not prime because $15 \div 3 = 5$. The number 21 is not prime because $21 \div 3 = 7$. This process can be repeated for every number not shown above.

Prime Factorization: To prime factorize a number, keep dividing by the smallest divisible prime number possible, until the division equal 1. Then, the prime factorization of the original number is all of those divisible prime numbers multiplied (don't actually do the multiplication, just state them all with a multiplication sign between each number).

Example: What is the prime factorization of 180?

The number 180 is even, so the smallest prime number that can go into it is 2. So, $180 \div 2 = 90$. This answer is also even, $90 \div 2 = 45$. The number 45 is not divisible by 2, but can be divided by 3 (the next smallest prime number), so $45 \div 3 = 15$. This is also divisible by 3, so $15 \div 3 = 5$. Finally, $5 \div 5 = 1$. Since this answer is 1, we have finished. So, taking all of the primes we divided by, the prime factorization of 180 is:

$$180 = 2 \cdot 2 \cdot 3 \cdot 3 \cdot 5.$$

This can be easily checked by multiplying the prime factors and seeing that the answer is 180.

Basics of Set Theory

In order to talk about Classification of Real Numbers, we need to have an understanding of the notation used. But, before we can talk about the notation, we need to understand what a set is.

A set is a collection of things, a group. A set can be a group of numbers, of items, of people, of anything really. For Mathematics, it is most common for us to use numbers though, so that will be the main focus. You can name the sets, and whenever you mention the letter, you mean to say all of the items in the set. It is important to note that sets do not have repetition inside. If there is an item in the set, then there cannot be another of the exact same item in that same set. You have to have distinguishable items. There are a few ways of describing a set:

1st way:

$$\text{List: } S = \{3, 4, 10, 18, 29, 8, 23\} \quad \text{or} \quad H = \{3, 6, 9, 12, 15, 18, ...\}$$

The list method actually itemizes what is in the set. Each item (separated by a comma) is called an element of the set. The list method will either state all of the elements, or it will show a pattern so that the reader can find the rest of the elements, as seen in set H as it contains all multiples of 3.

2nd way:

$$\text{Set Builder: } G = \{3 \cdot x + 4 \mid x \in S\}$$

The set builder method describes a rule for the elements, and the rule has a condition on it. The rule is found before the vertical bar, while the condition is after. A lot of the time, you will see the symbol $\in$. This stands for "is an element of". So, in our set G, the variable x is an element of S, or the set from the first example (the set $\{3, 4, 10, 18, 29, 8, 23\}$). This mean you use the rule for each of the elements in S to find all of the elements of G. Thus:

$$G = \{3 \cdot (3) + 4,\ \ 3 \cdot (4) + 4,\ \ 3 \cdot (10) + 4,\ \ 3 \cdot (18) + 4,\ \ 3 \cdot (29) + 4,\ \ 3 \cdot (8) + 4,\ \ 3 \cdot (23) + 4\}$$
$$G = \{13, 16, 34, 58, 91, 28, 73\}$$

3rd way:
The last way to describe a set is just to describe the set using words. For instance, set J is the set of all numbers ending with 1. Another example is set M is the set of all pens of the world. As you see, both are just described in words, but the set J can also be described as a list and as a set builder. Set M can also be described using a list, but it would be a lot harder. When making sets of anything besides numbers, you will always use the list method or the descriptive method.

Set Arithmetic
There are some operations that we can do with sets. The most basic operations is the union and the intersection. These are done between two sets, just like addition and subtraction are done between two numbers. The result of the union and the intersection is a single, new set, just like the result of addition and subtraction is a single, new number.

Union
The union, or $\cup$, is an operation that is like addition of sets. When you take a union of two sets, you are taking all of the elements in the first and taking all of the elements in the second, and putting them into a new set. Remember, you do not repeat elements inside a set, as seen below in the last example.

Examples:

$$\{2,3,5\} \cup \{1,6,7\} = \{1,2,3,5,6,7\}$$
$$\{1,3,5,7,9,11,...\} \cup \{0,2,4,6,8,10,...\} = \{0,1,2,3,4,5,6,7,8,9,10,11,...\}$$
$$\{15,30,45,60\} \cup \{10,20,30,40,50,60\} = \{10,15,20,30,40,45,50,60\}$$

Intersection
The intersection, or $\cap$, is an operation that is like subtraction of sets. When you take an intersection of two sets, you are taking only the elements that are in both sets, and putting them into a new set. As always, do not repeat elements.
Examples:

$$\{1,3,5,7,9\} \cap \{2,3,4,5,6,7,8\} = \{3,5,7\}$$
$$\{..., -5, -3, -1, 1, 3, 5, ...\} \cap \{..., -15, -10, -5, 0, 5, 10, 15, ...\} = \{..., -25, -15, -5, 5, 15, 25, ...\}$$
$$\{-8, -4, 2, 5, 19, 22\} \cap \{-3, -2, -1, 0, 1, 2, 3\} = \{2\}$$

Empty Set
In mathematics, 0 is a key number for many operations. It gives us a mathematical sense of nothing. This is also true for set theory, except the concept of 0 is called the empty set ($\varnothing$). The empty set is a set with no elements in it:

$$\text{Empty Set: } \varnothing = \{\}$$

Make sure you remember that the empty set is not $\{\varnothing\}$. This is a completely different thing, and is specifically used in certain situations. The empty set, $\varnothing$, is the set containing nothing, while $\{\varnothing\}$ is the set containing the set that contains nothing. Very different.

Examples of the empty set in action:

$$\{3,4,5,6\} \cup \varnothing = \{3,4,5,6\}$$
$$\{2,3,4\} \cap \{9,10,11\} = \varnothing$$

Classification of Real Numbers

There are many different ways to group members within the whole Real Numbers system and each grouping is called a subset. Some of the most important subsets are the ones we use to classify the Real Numbers. Each has a special symbol for a quick and understandable reference as these subsets are commonly used. There are 5 main subsets.

Natural (Counting) Numbers: $\mathbb{N} = \{1, 2, 3, 4, 5, \ldots\}$

Whole Numbers: $\mathbb{W} = \{0, 1, 2, 3, 4, 5, \ldots\}$

Integers: $\mathbb{Z} = \{\ldots, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, \ldots\}$

Rational Numbers: $\mathbb{Q} = \{\frac{a}{b} \mid \text{a,b} \in \mathbb{Z}\}$
These numbers can always be written as a decimal that terminates or keeps repeating the same pattern.

Irrational Numbers: $\mathbb{I} = \{\text{x} \mid \text{x is an infinite decimal that does not continuously repeat}\}$
These numbers can not be written as a fraction like the Rational Numbers can.

With these set being defined, the Real Numbers can be defined as:
Real Numbers: $\mathbb{R} = \mathbb{Q} \cup \mathbb{I}$
The Real Numbers are all the numbers in the Rational and Irrational sets.

Parity of Numbers
The numbers within the Integer subset can be classified further into having one of two parities, either even or odd. Even parity numbers are numbers that, when divided by 2, result in an integer. Odd parity numbers are the rest of the integers. Example:

$$10 \div 2 = 5, \qquad 10 \text{ is even}$$
$$9 \div 2 = 4.5, \qquad 9 \text{ is odd}$$

Even numbers: $\{..., -6, -4, -2, 0, 2, 4, 6, ...\} = \{2 \cdot x \mid x \in \mathbb{Z}\}$
Odd numbers: $\{..., -5, -3, -1, 1, 3, 5, ...\} = \{2 \cdot x + 1 \mid x \in \mathbb{Z}\}$

Least Common Multiple and Greatest Common Factor

Least Common Multiple (LCM)
The multiples of a number is that number multiplied by the Natural Numbers, or 1, 2, 3, 4, 5, . . . So, for example, the multiples of 3 and 6 are:

$$3 \cdot 1,\ \ 3 \cdot 2,\ \ 3 \cdot 3,\ \ 3 \cdot 4,\ \ 3 \cdot 5, ... = 3, 6, 9, 12, 15, ...$$
$$6 \cdot 1,\ \ 6 \cdot 2,\ \ 6 \cdot 3,\ \ 6 \cdot 4,\ \ 6 \cdot 5, ... = 6, 12, 18, 24, 30, ...$$

Now, the least common multiple (LCM) is a number that is the smallest multiple in common of two or more numbers. In the case of 3 and 6 above, 6 is the LCM. You can easily find the LCM by listing out a fair number of multiples of each number, then beginning from the left side, and, starting with the smallest multiple, check if that particular multiple is in all of the lists. If the first multiple is not common in each list, then look at the next multiple. Keep doing this until you find the first multiple that is common in each list. This is the least common multiple.

Greatest Common Factor (GCF)
The factors of a number is all of the Whole Numbers that, when multiplied by another Whole Number, give the original number back. Normally, you would not be looking for factors of negative numbers (as you can just take the negative out as a factor of -1), and you can't find factors of numbers besides Integers. For instance, the factors of 56 are: 1, 2, 4, 7, 8, 14, 28, and 56. These can be checked pretty easily, as:

$$1 \cdot 56 = 56$$
$$2 \cdot 28 = 56$$
$$4 \cdot 14 = 56$$
$$7 \cdot 8 = 56$$

Factors will always be Whole Numbers, and you will always multiply a factor by another factor to get the original number, so that means you will always have an even number of factors (unless looking at 1 or a square–these result in an odd number of factors where the center factor is the square root).

So, the GCF is a number that is the biggest factor in common of two or more numbers. The first step to finding the GCF is to find all of the factors of each of the numbers, then begin at the right (instead of the left in LCM) and start with the largest number. Check each list to see if it is a common factor, if not, then move on to the next highest factor in all of the lists. If there is no number greater than 1 that is common in each of the lists, then 1 is the GCF.
Example:

$$56 : 1, 2, 4, 7, 8, 14, 28, 56$$
$$80 : 1, 2, 4, 5, 8, 10, 16, 20, 40, 80$$
$$8 \text{ is the GCF}$$

Fraction Arithmetic

Simplifying (Reducing) Fractions:

1st way:

Prime factorize numerator and denominator. For each one to one occurrence of a number in the top and bottom, those two numbers cancel. Meaning, they can be divided out and you would still have an equivalent fraction.

Example 1:

$$\frac{100}{120} = \frac{2 \cdot 2 \cdot 5 \cdot 5}{2 \cdot 2 \cdot 2 \cdot 3 \cdot 5} = \frac{2}{2} \cdot \frac{2}{2} \cdot \frac{5}{5} \cdot \frac{5}{2 \cdot 3} = 1 \cdot 1 \cdot 1 \cdot \frac{5}{6} = \frac{5}{6}.$$

As you could see, after dividing out all common prime factors you must multiply the remaining numbers in the numerator and the denominator. So, the fraction $\frac{5}{6}$ is $\frac{100}{120}$ most simplified form.

Example 2:

$$\frac{300}{30} = \frac{2 \cdot 2 \cdot 3 \cdot 5 \cdot 5}{2 \cdot 3 \cdot 5} = \frac{2}{2} \cdot \frac{3}{3} \cdot \frac{5}{5} \cdot \frac{2 \cdot 5}{1} = 1 \cdot 1 \cdot 1 \cdot \frac{10}{1} = 10$$

2nd way:

Find a common factor in the top and bottom, one that you can see easily. Then, divide both the numerator and the denominator by this common factor. Keep doing this until there is no common factor between the top and the bottom.

Example 1:

$$\frac{100}{120} = \frac{100 \div 10}{120 \div 10} = \frac{10}{12} = \frac{10 \div 2}{12 \div 2} = \frac{5}{6}$$

Since both the numerator and denominator numbers had a zero on the end, there must be a factor of 10 in each, dividing this out resulting in a fraction with a factor of 2 in the top and bottom. Dividing the 2 out then gives us a fraction with no common factor in the top and bottom, so this must be the most simplified form of the fraction.

Example 2:

$$\frac{300}{30} = \frac{300 \div 30}{30 \div 30} = \frac{10}{1} = 10$$

Conclusion:

Between the two methods, the first is the more rigorous way. It will always give you the most simplified fraction at the end. The only issue is that you must prime factorize the numbers, which can be a problem when working with large numbers. The 2nd method works well for large numbers, but you have to keep an eye out for any common factors. If there is one common factor, no matter how small, then it is not in simplest form.

What seems to work the best is to do a little of both. For large numbers or very easy to see factors (such as a 0 on the end of both numbers) find the common factor and divide it out. If you get stuck with that, then prime factorize the numbers and divide out common primes.

Multiplying Fractions:

To multiply fractions, all you need to do is multiply straight across the numerators and denominators, and then reduce the resulting fraction:

$$\frac{5}{6} \cdot \frac{4}{3} = \frac{5 \cdot 4}{6 \cdot 3} = \frac{20}{18} = \frac{20 \div 2}{18 \div 2} = \frac{10}{9}$$

$$\frac{2}{8} \cdot \frac{8}{4} = \frac{2 \cdot 8}{8 \cdot 4} = \frac{16}{32} = \frac{16 \div 16}{32 \div 16} = \frac{1}{2}$$

It can be useful to simplify the fractions before doing the multiplication. To do this, you can first simplify each fraction individually, but you can actually simplify them before fully multiplying the numerator and denominator. Look for a common factor in the numerator of the first fraction and the denominator of the second fraction, then divide it out of them. Keep doing this until there are no more common factors. Then, do the same with the denominator of the first fraction and the numerator of the second.

$$\frac{5}{6} \cdot \frac{4}{3} = \frac{5}{6 \div 2} \cdot \frac{4 \div 2}{3} = \frac{5}{3} \cdot \frac{2}{3} = \frac{5 \cdot 2}{3 \cdot 3} = \frac{10}{9}$$

$$\frac{2}{8} \cdot \frac{8}{4} = \frac{2 \div 2}{8} \cdot \frac{8}{4 \div 2} = \frac{1}{8} \cdot \frac{8}{2} = \frac{1}{8 \div 8} \cdot \frac{8 \div 8}{2} = \frac{1}{1} \cdot \frac{1}{2} = \frac{1}{2}$$

To do this the easiest way, try simplifying the individual fractions first, then simplify before multiplying, and then finally multiply it out. There should not be any more simplification needed after all of that.

Dividing Fractions:
To divide fractions, you need to take the first fraction and multiply it by the reciprocal (or flipped) of the second fraction.

$$\frac{4}{11} \div \frac{5}{11} = \frac{4}{11} \cdot \frac{11}{5} = \frac{4}{11 \div 11} \cdot \frac{11 \div 11}{5} = \frac{4}{1} \cdot \frac{1}{5} = \frac{4}{5}$$

Compound fractions (below) are common in higher mathematics, but you treat them the same as the problem just shown, multiply by the reciprocal of the bottom fraction.

$$\frac{\frac{9}{10}}{\frac{3}{16}} = \frac{9}{10} \cdot \frac{16}{3} = \frac{9 \div 3}{10} \cdot \frac{16}{3 \div 3} = \frac{3}{10 \div 2} \cdot \frac{16 \div 2}{1} = \frac{3}{5} \cdot \frac{8}{1} = \frac{24}{5}$$

Adding Fractions:
Like Denominator: You must have the same denominators in both fractions to add them; there is no other way to do it (unless you convert to decimals first). If you do in fact have like denominators, then all you need to do is add the tops and keep the denominator on bottom:

$$\frac{5}{3} + \frac{4}{3} = \frac{5+4}{3} = \frac{9}{3} = 3$$

Always make sure to simplify the resulting answer.

Unlike Denominator: If you want to add two fractions with different denominators, then you must make it the same denominator. The easiest ways to go about this are either multiplying by the other denominator or multiplying to make the Least Common Multiple (LCM).

Multiplying by other denominator:

$$\frac{7}{6} + \frac{1}{4} = \frac{4}{4} \cdot \frac{7}{6} + \frac{1}{4} \cdot \frac{6}{6} = \frac{28}{24} + \frac{6}{24} = \frac{34}{24} = \frac{17}{12}$$

Multiply by LCM: Find LCM of both of the denominators, then multiply by a factor that makes each denominator that number:

$$\frac{7}{6} + \frac{1}{4} = \frac{2}{2} \cdot \frac{7}{6} + \frac{1}{4} \cdot \frac{3}{3} = \frac{14}{12} + \frac{3}{12} = \frac{17}{12}$$

Doing the LCM method gets to the most simplified form the fastest, but requires more work up front. Multiplying by the other denominator is the simplest to see, but can require a lot more work later.

Subtracting Fractions:
Subtracting fractions is the exact same process as adding fractions, except you must subtract the numerators instead of adding them.

Converting between Fractions, Decimals, and Percents

Converting from Fractions to Decimals
To convert from fractions to decimals, use long division to divide the numerator by the denominator.

Converting from Decimals to Fractions
To convert from decimals to fractions, you must notice a couple things first. If the decimal never repeats a pattern infinitely, then you cannot convert the decimal to a fraction. If, at any point, the decimal starts to repeat a pattern infinitely or comes to a stop (or terminates), then you can change the decimal into a fraction. Follow the instructions for each case of decimal:

Terminating Decimals
To change a terminating decimal into a fraction, move the decimal all the way to the very right end of the number, counting as you go along. Then, you must divide the new number by 10 to the power of the number of digits you passed. Finally, simplify the resulting fraction.
Example:

$$84.6785 \to 846785. \quad \text{The decimal passed 4 digits, so,}$$
$$84.6785 = \frac{846785}{10^4} = \frac{846785}{10000} = \frac{169537}{2000}.$$

Infinitely Repeating Decimals
To change an infinitely repeating decimal into a fraction, you need to do a little trick to it. First, make x equal to your decimal. Then, find out for how many digits the pattern repeats. Create a new equation by multiplying the x by 10 to the power of the number of digits repeated and making it equal to the decimal multiplied by the same power of 10. If you subtract this new equation by the equation x equal to the decimal, the repeating pattern should cancel out, and you can solve easily for x. Finally, if there still is a terminating decimal in the numerator, multiply and divide by the same factor of 10 such that the decimal cancels, and then simplify.
Example 1:

$$\begin{aligned} x &= 0.33333333... = 0.\overline{3} \quad \text{only 1 digit is repeated, so,} \\ 10^1 \cdot x &= 10^1 \cdot 0.\overline{3} \quad \text{which is,} \\ 10x &= 3.\overline{3}. \end{aligned}$$

Doing the subtraction gives:

$$\begin{array}{rl} 10x & = 3.\overline{3} \\ - \quad x & = 0.\overline{3} \\ \hline 9x & = 3 \end{array}$$

Solving for x gives $x = \frac{3}{9} = \frac{1}{3}$.

Example 2:

$$\begin{aligned} x &= 27.12758585858... = 27.127\overline{58} \quad \text{2 digits are repeated, so,} \\ 10^2 \cdot x &= 10^2 \cdot 27.127\overline{58} \quad \text{which is,} \\ 100x &= 2712.7\overline{58}. \end{aligned}$$

Doing the subtraction gives:

$$\begin{array}{rcl} 100x & = & 2712.758\overline{58} \\ - \quad x & = & 27.127\overline{58} \\ \hline 99x & = & 2685.631 \end{array}$$

Solving for x gives $x = \frac{2685.631}{99}$. However, I haven't yet converted the number. I need to remove the decimal out of the numerator to fully convert the decimal. The last step is to multiply top and bottom (to keep the numbers equal) by 10 to the power of the number of digits to the right of the decimal. In this case, I need to multiply by $10^3 = 1000$. Doing so,

$$\frac{2685.631}{99} \cdot \frac{1000}{1000} = \frac{2685631}{99000}. \quad \text{This is the simplest form.}$$

Converting from Decimals to Percents
To convert from decimals to percents, multiply the decimal by 100, and add a % on the end.

Converting from Percents to Decimals
To convert from percents to decimals, just divide by 100 and take off the %.

Converting from Percents to Fractions
To convert from percents to fractions, convert the percent to a decimal, then convert the decimal to a fraction.

Converting from Fractions to Percents
To convert from fractions to percents, first convert the fraction to a decimal, then convert the decimal to a percent.

Basic Units of Measure

We measure things to describe the natural world. So, what are things we need to know? It can be said that there are only 7 things that are fundamental in order to describe all the different parts of nature, and those seven are length (or distance), mass, time, temperature, electric current, amount of light, and an amount of a substance. There is a problem though, the United States utilizes a different system of units than most of the rest of the world. The main system that is used is the International System of Units (SI for short). Therefore, we need to know how to convert between the different systems. But first, here is the 7 fundamental measurements and their units:

	American	SI
Length	Feet (ft)	Meters (m)
Mass	Pounds (lbs)	Kilograms (kg)
Time	Seconds (s)	same
Temperature	Fahrenheit (°F)	Kelvin (K)
Electric Current	Ampere (A)	same
Amount of Light	Candela (cd)	same
Amount of a Substance	Mole (mol)	same

There is some caution that must be noted with this list:

- Most of the time, other countries use Celsius (°C) as their unit of measure for temperature, but Kelvin is the unit that scientists use.
- Electric current is measured in amperes (amps for short), but an Amp is based on the amount of charge (an electron charge, e) in an area over time.
- The SI unit of mass is kilogram, which is actually a measure of amount of matter, while the pound is the measure of Earth's gravity affecting an amount of matter, or weight. What this means is that your weight will change from planet to planet, since each has a different force of gravity, but your mass will not change.

Even though there are only 7 fundamental units, there are many, many things in the natural world that need to be described and measured. We measure these things by multiplying and dividing different combinations of the fundamental units. The other measurements we will cover here is area, volume, speed, as well as more units of length and mass.

Length:
There are different units of measure for length because there are different distances ranging from smaller than microscopic to larger than astronomical. Here are the most commonly used units of the American and SI system:

American	SI
12 inches (in) = 1 ft	1000 millimeters (mm) = 1 m
1 yard (yd) = 3 ft	100 centimeters (cm) = 1 m
1 mile (mi) = 5280 ft	1 kilometer (km) = 1000 m

Mass:
The most common mass measurements are lbs for the American and kg for the SI, but there are two smaller units that can be easier to use sometimes, and that is the ounce (oz) and gram (g).

$$16 \text{ oz} = 1 \text{ lb} \qquad 1000 \text{ g} = 1 \text{ kg}$$

Area:

While length is the measure of a one dimensional space, area is the measure of a two dimensional space. A basic example of area is the area of a rectangle, which is equal to the measure of its one dimensional length multiplied by the measure of its one dimensional width. Not only do the measurements get multiplied, but the units get multiplied as well. It is important to use the same unit of measure when doing math. So, if our rectangle was 6 ft long and 5 ft wide, then its area is 30 ft^2 read as 30 feet squared.

There is generally no change in unit (besides the square on the unit) for area, but it is important to know that area unit conversions result in bigger changes than regular length measures, which you will see below.

American	SI
144 in^2 = 1 ft^2	1,000,000 mm^2 = 1 m^2
1 yd^2 = 9 ft^2	10,000 cm^2 = 1 m^2
1 acre = 43,560 ft^2	1 km^2 = 1,000,000 m^2

The unit mi^2 is not used very much in the American units, but the acre is used frequently for plots of land. But, for your information, 1 mi^2 = 27,878,400 ft^2.

Volume

Volume is an interesting measurement, as there are a lot of different units. The reason for this is that the physical world we live in is three dimensional, so that is the most frequent type of object we use. For instance, in cooking there are about 7 commonly used units! But, we will not be covering all of this. We are only considering the most commonly used volume measurements in math and science.

American	SI
1728 in^3 = 1 ft^3	1,000,000,000 mm^3 = 1 m^3
1 yd^3 = 27 ft^3	1,000,000 cm^3 = 1 m^3
1 acre foot = 43,560 ft^3	1 km^3 = 1,000,000,000 m^3
1 (liquid) gal = 231 in^3	1 cm^3 = 1 mL
1 (dry) gal = 268.8 in^3	1 liter (L) = 1000 mL

Though the acre foot is not used frequently by the general public, it would be the biggest commonly used measure for the American system. A mile cubed is just too big of a number. Also, the SI system uses liters and milliliters the most, but knowing the meter conversions will help when trying to find how much liquid a solid shape can hold.

Speed:

Speed is the measurement of a distance changing over time. Speed tells us how fast we were moving by dividing the distance traveled by the time traveled. Time is measured in both seconds (s) and hours (hr).

$$1\ \frac{\text{mi}}{\text{hr}} = 1.467\ \frac{\text{ft}}{\text{s}} \qquad 3.6\ \frac{\text{km}}{\text{hr}} = 1\ \frac{\text{m}}{\text{s}}$$

Conversions:

Now that you have been introduced to the most common units used in the American and SI systems, we need to show you how to convert between them. This is done with a conversion ratio, which there is generally one ratio for each type of measurement (length, area, volume, etc.). Here are the ratios:

Type of Unit	Conversion Ratio
Length	3.281 ft = 1 m
Mass	2.205 lbs = 1 kg
Area	use length measurement
Volume	use length measurement
Speed	use length measurement and 3600 s = 1 hr

A simple example of a unit conversion is to change inches to meters. The conversion we have above is for feet to meters, so the only way we can use this ratio is if we first change the inches to feet (ratio given above). So, what we do is place every ratio we need as fractions that are being multiplied. We need to place every ratio in a way that cancels the units we do not want, the way this happens depends on the original measurement. So, let's convert 50 inches to meters:

$$50 \text{ in} = \frac{50 \text{ in}}{1} \cdot \frac{1 \text{ ft}}{12 \text{ in}} \cdot \frac{1 \text{ m}}{3.281 \text{ ft}} = \frac{50 \cdot 1 \cdot 1 \text{ in} \cdot \text{ft} \cdot \text{m}}{1 \cdot 12 \cdot 3.281 \text{ in} \cdot \text{ft}} = \frac{50 \cancel{\text{in}} \cdot \cancel{\text{ft}} \cdot \text{m}}{39.372 \cancel{\text{in}} \cdot \cancel{\text{ft}}} = \frac{50}{39.372} \text{ m} = 1.27 \text{ m}$$

As you can see, when you multiplied the fractions together, you get to cancel common units on the top and bottom, and in the end, you should always have the unit you desire, in the case above, meters. Now, we will go over another example, this time from one speed to another: from 10 meters per second (meters divided by seconds) to feet per hour.

$$\begin{aligned} 10 \frac{\text{m}}{\text{s}} &= \frac{10 \text{ m}}{1 \text{ s}} \cdot \frac{3.281 \text{ ft}}{1 \text{ m}} \cdot \frac{60 \text{ s}}{1 \text{ min}} \cdot \frac{60 \text{ min}}{1 \text{ hr}} \\ &= \frac{10 \cdot 3.281 \cdot 60 \cdot 60 \text{ m} \cdot \text{ft} \cdot \text{s} \cdot \text{min}}{1 \cdot 1 \cdot 1 \cdot 1 \text{ s} \cdot \text{m} \cdot \text{min} \cdot \text{hr}} = \frac{118116 \cancel{\text{m}} \cdot \text{ft} \cdot \cancel{\text{s}} \cdot \cancel{\text{min}}}{1 \cancel{\text{s}} \cdot \cancel{\text{m}} \cdot \cancel{\text{min}} \cdot \text{hr}} = 118116 \frac{\text{ft}}{\text{hr}} \end{aligned}$$

What is important is that you always make sure you do whatever multiplication you need to get the desired unit, as shown above we needed to convert to minutes and then to hours. Still, we always need to multiply the ratios in a way that will cancel the units, one is above the fraction bar and the same is below.

Properties of Exponents

Exponents:
Exponents have a base and a power. If you see this: a^b, then a is the base and b is the power. The expression a^b means take the number a and multiply it by itself b times. Example:

$$2^3 = 2 \cdot 2 \cdot 2 = 8$$
$$4^5 = 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 = 1024$$

Properties of Exponents:

$$a^m \cdot a^p = a^{m+p}$$
$$(a^n)^o = a^{n \cdot o}$$
$$\frac{b^q}{b^r} = b^{q-r}$$
$$c^{-d} = \frac{1}{c^d}$$
$$x^0 = 1$$
$$y^1 = y$$
$$(i \cdot j)^k = i^k \cdot j^k$$
$$(\frac{i}{j})^k = \frac{i^k}{j^k}$$
$$m^{\frac{n}{o}} = \sqrt[o]{m^n} = (\sqrt[o]{m})^n$$

Squared and Cubed Tables

Number	Squared	Cubed
1	1	1
2	4	8
3	9	27
4	16	64
5	25	125
6	36	216
7	49	343
8	64	512
9	81	729
10	100	1000
11	121	1331
12	144	1728
13	169	2197
14	196	2744
15	225	3375
16	256	4096
17	289	4913
18	324	5832
19	361	6859
20	400	8000

Properties of Logarithms

Logarithms:
Logarithms have a base and an argument. If you see this: $\log_a(b)$, then a is the base of the logarithm and b is the argument. The expression $\log_a(b)$ is asking what exponential power, for example c, can you put on a such that $a^c = b$. Logarithms are the inverse functions of exponential functions with the variable in the power (unlike radicals where the variable is in the base). Examples:

$$\log_2(8) = 3$$
$$\log_4(1024) = 5$$

Properties of Logarithms:
First and foremost, you cannot take the logarithm of a negative number, no matter the base.

$$log_a(x \cdot y) = log_a(x) + log_a(y)$$
$$log_b(\frac{f}{g}) = log_b(f) - log_b(g)$$
$$log_c(z^w) = w \cdot log_c(z)$$
$$log_p(o) = \frac{log_m(o)}{log_m(p)}$$
$$log_n(n) = 1$$
$$log_n(1) = 0$$

Properties of Radicals

Radicals:
Radicals are mathematical statements that involve roots. The most common root used is called the square root, and it is written short hand as $\sqrt{a}$, but the long version is written $\sqrt[2]{a}$. For the square root, 2 is the degree. There are many types of roots than just the square root, and they are either written as $\sqrt[x]{y}$ or $y^{\frac{1}{x}}$, and it is said that the x is the degree of the root. What radicals mean is if $\sqrt[a]{b} = c$, then $c^a = b$. Radicals are the inverse functions of exponential functions where the variable is in the base (unlike logarithms where the variable is in the power).
Examples:

$$\sqrt{4} = \sqrt[2]{4} = \pm 2$$
$$\sqrt[3]{64} = 4$$

Properties of Radicals:
-If you take an even degree root of a positive number, you will get a Real Number.
-If you take an even degree root of a negative number, you will get a Complex Number, not a Real Number.
-If you take an odd degree root, no matter the sign on the number, you will get a Real Number.

$$(\sqrt[c]{b})^a = \sqrt[c]{b^a} = b^{\frac{a}{c}}$$
$$\sqrt[a]{b} \cdot \sqrt[a]{c} = \sqrt[a]{b \cdot c}$$
$$d\sqrt[a]{b} \cdot e\sqrt[a]{c} = d \cdot e \cdot \sqrt[a]{b \cdot c}$$
$$g\sqrt[x]{y} + h\sqrt[x]{y} = (g+h) \cdot \sqrt[x]{y}$$
$$\sqrt[m]{\sqrt[n]{z}} = \sqrt[(m \cdot n)]{z}$$
$$\sqrt[a]{1} = 1$$
$$\sqrt[b]{0} = 0$$

It is important to note that there are some differences between the answer to even and odd square roots. The first way to see this is to take the square and cube of both 2 and -2:

Square:
$$2^2 = 2 \cdot 2 = 4$$
$$(-2)^2 = -2 \cdot -2 = 4$$
Cube:
$$2^3 = 2 \cdot 2 \cdot 2 = 8$$
$$(-2)^3 = -2 \cdot -2 \cdot -2 = -8$$

As you can see, when you square (or take any even power for that matter) a number and the negative of that number, you get the exact same answer. However, when you cube (or take any odd power) a number and the negative of that number, you get two distinct answers. So, when taking a square root (or even root), you are asking, "What number must I square to get this number?" Well, there are two numbers that can do the job: the number and its negative. We write this with a $\pm$ in front of the number, take the positive number as an answer and take the negative number as an answer. If you take an odd root, you only get one number as a result, and it will always have the same sign as the number that you are rooting.

Basics of Algebra

The first thing we need to define in order to understand Algebra is a variable. A variable is generally used to denote an unknown value, and you normally use a letter to describe that unknown. Variables can actually be one of two things, a constant or a changing value.

Constant vs. Changing Value:
Numbers can be called constants, or in other words they do not change their value. The number 3 has the value 3. It can never change its value, and thus is constantly worth 3 units. Now, let's say we had a variable x. What is its value? We don't know the exact value, but if someone tells us that x is a constant, then we know that it can only have one value. However, if someone told us that x is not a constant variable, then x can take on many, many different values, and we don't know what it is at present. It is important, though, that with any variable you can treat it as a number, meaning you can add variables, subtract variables, multiply and divide variables, but you just cannot find the actual answer to that operation.

Variables in Expressions and Equations:
When a variable is sitting right next to another variable or number, it means that the variable is being multiplied with the variable or number ($10 \cdot z = 10z, xy = x \cdot y$). The number 10 is called the coefficient of the variable, and every variable has a coefficient. If there is no number next to a variable, then it is understood that its coefficient is 1 ($m = 1 \cdot m$). Here are some examples of this fact and variables in action:

$$3x, \quad 4xy, \quad a^2 + 6b, \quad \frac{56x + 17y}{x^8 - z}, \quad a^m = b^n + c^{o-p}, \quad \text{etc.}$$

Combining Like Terms:
In these examples we cannot reduce the expressions and equations into any simpler statements, but there are times when you can. Variables can be added and subtracted when they are "like terms". This means that every term, or group of numbers and variables that are being multiplied and divided, that have the **exact** same variables can have their coefficients added or subtracted. This process is called combining like terms.

$3x - 7x + 2y$	Combine like terms.
$\underline{3x} + \underline{-7x} + 2y$	Here are two.
$3 + -7 = -4$	Adding coefficients gives -4, producing,
$-4x + 2y$	No other like terms.

Note that we changed the $3x - 7x$ into $3x + -7x$. This is very common practice, and it keeps the negative with the coefficient even if you move the number around by the commutative property.

$6a - 6b + 12ab - 14b + a$	Combine like terms.
$\underline{6a} - 6b + 12ab - 14b + \underline{a}$	Here are two.
$6 + 1 = 7$	Adding the coefficients gives 7, producing,
$7a - 6b + 12ab - 14b$	There are still more like terms.
$7a + \underline{-6b} + 12ab + \underline{-14b}$	Here they are.
$-6 + -14 = -20$	Adding the coefficients gives -20, producing,
$7a - 20b + 12ab$	No other like terms.

As you can see, you need to combine all like terms possible to find the simplest form of the expression or equation. Plus, combining all terms that you can will often make the expression or equation much more organized and easier to handle. It is just good practice to always combine terms as much as you can.

Substitution of Value:

If you were given this expression $3x + \frac{15}{x}$, could you find the exact value of it? No. As stated above, you do not know the value of the variable x, thus you cannot do the operations. But, what if you were then given that $x = 3$? Now, we know what the value of x is, and since x is the only variable in the expression, that means we can solve it! How do we do this? For every x you see in the expression, substitute a 3 there instead. Commonly this is said as "plugging in" a 3 into the expression.

$$\begin{aligned} 3x + \frac{15}{x} \quad & \text{We know that } x = 3\text{, so we plug it in.} \\ 3(3) + \frac{15}{(3)} \quad & \text{We put the 3 in parentheses to show we plugged it in.} \\ 3 \cdot 3 + \frac{15}{3} \quad & \text{Now we just solve the expression.} \\ 9 + 5 \quad & \text{Adding gives our answer of 14.} \end{aligned}$$

By plugging in all variables, it turns the variable expression into one of only numbers, which can be simplified. You use the exact same process when confronted with even more complex problems.

Quick tip, if you have an expression or equation that has like terms, it is often easier to combine like terms first and then substitute the numbers in rather than doing the substitution first.